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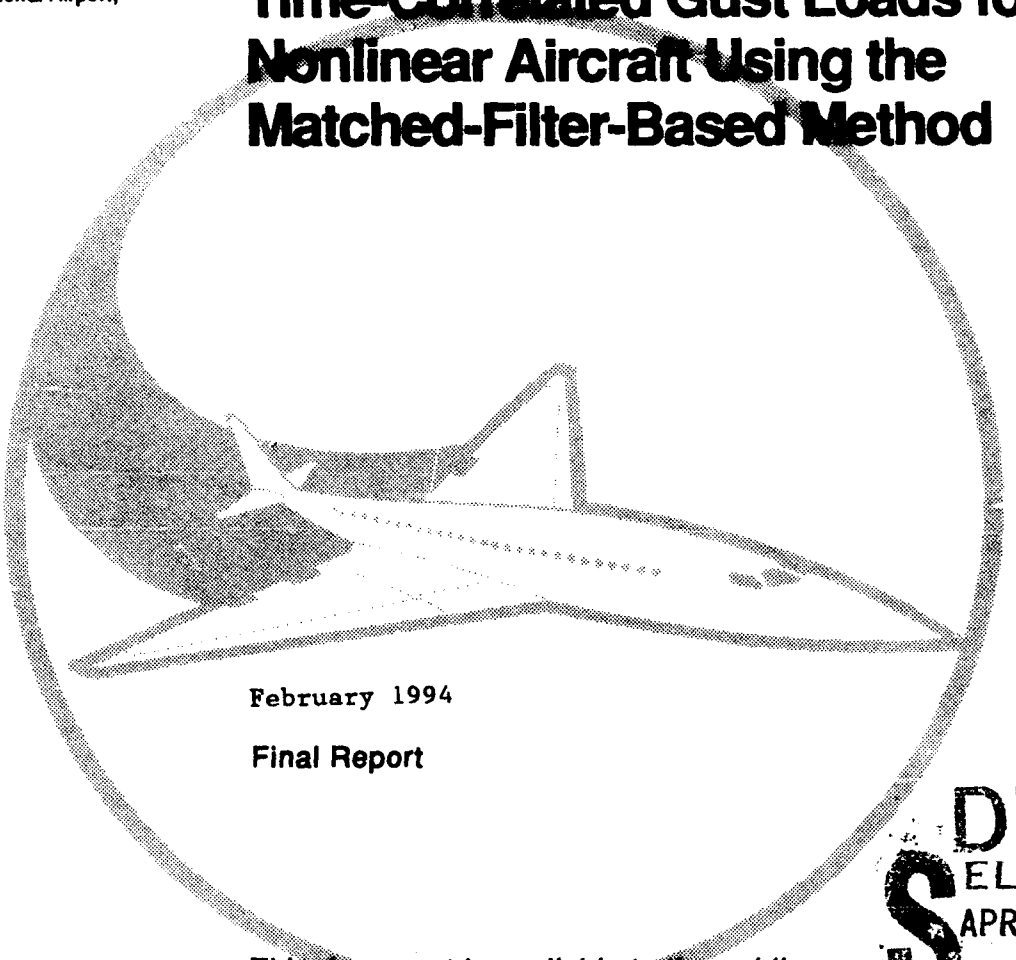
A Computer Program to Obtain Time-Correlated Gust Loads for Nonlinear Aircraft Using the Matched-Filter-Based Method



February 1994

Final Report

U.S. Department of Transportation
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16. Abstract

NASA Langley Research Center has, for several years, conducted research in the area of time-correlated gust loads for linear and nonlinear aircraft. The results of this work led NASA to recommend that the Matched-Filter-Based One-Dimensional Search Method be used for gust load analyses of nonlinear aircraft. This manual describes this method, describes a FORTRAN code which performs this method, and presents example calculations for a sample nonlinear aircraft model. The name of the code is MFB1DS (Matched-Filter-Based One-Dimensional Search). The program source code, the example aircraft equations of motion, a sample input file, and a sample program output are all listed in the appendices.

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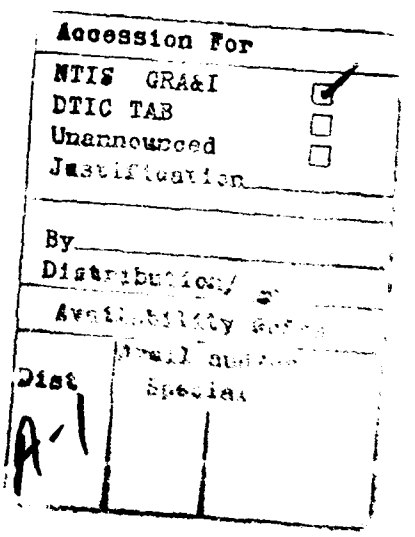

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EXECUTIVE SUMMARY

This report was sponsored under the Federal Aviation Administration's Aircraft Catastrophic Failure Prevention Research Program, Mission Need Statement No. 066-110. This report documents the NASA recommended Matched-Filter-Based One-Dimensional Search Method for the gust load analysis of nonlinear aircraft. The FAA Technical Program Monitor was Mr. Terry Barnes, ANM-105N, National Resource Specialist, Flight Loads and Aeroelasticity: FAA COTR was Mr. Thomas DeFiore, ACD-220, Principal Investigator, Flight Loads.

INTRODUCTION

This manual and the accompanying code were developed as partial fulfillment of a NASA agreement with the FAA. The agreement had three main tasks. First, two NASA developed gust analysis methods were to be brought to the same level of maturity. These analysis methods were the Matched-Filter-Based (MFB) method and the Stochastic-Simulation-Based (SSB) method. Upon completion of the development work, the second task was to compare the methods and make a recommendation selecting which approach was best suited for nonlinear analyses. This work was completed and the results given in a presentation at the Gust Specialists Meeting in LaJolla, California on April 22, 1993. At the meeting, it was recommended by NASA that the MFB one-dimensional search was the method of choice for time-correlated gust loads analysis of aircraft with nonlinear systems. The third and final task was to develop a transportable computer program and accompanying documentation for using the recommended method.

This manual describes a computer code called "MFB1DS" which performs the Matched-Filter-Based one-dimensional search. The MFB one-dimensional search is a deterministic method for obtaining maximized and time-correlated design loads for aircraft with nonlinearities. This method can, however, be applied to both linear and nonlinear aircraft. The paper summarizes the method, discusses the selection of gust intensity for the method, describes the FORTRAN code MFB1DS that performs the calculations, and presents numerical results for an example aircraft.

BACKGROUND

With the advent of aircraft that contain large numbers of nonlinearities in their flight control systems and/or gust load alleviation systems, existing methods for certifying aircraft for gust loads may not be adequate. For several years NASA Langley Research Center has conducted research in the area of time correlated gust loads and has published a number of papers on the subject (refs. 1-6). The initial research was restricted to mathematically linear systems (refs. 1-3). Recently, however, the focus of the research has been on defining methods that will compute design gust loads for an airplane with a nonlinear control system (refs. 4-6). To date, two such methods have been defined: one is based on matched filter theory; the other is based on stochastic simulation.

The Matched-Filter-Based (MFB) method was developed first and was reported on in reference 4. The MFB method employs optimization to solve for its answers and this method comes in two varieties: the first uses a one-dimensional search procedure and the second a multi-dimensional search procedure. The first is significantly faster to run and, based on experience with a number of nonlinear models gives design loads only slightly lower in magnitude than the second.

The Stochastic-Simulation-Based (SSB) method has evolved over the past two years. References 5 and 6 describe the method. In reference 6 a comparison of the MFB and SSB methods was made. The results predicted by the two methods are strikingly similar and demonstrate that the key quantities from the MFB method (viz. critical gust profile, maximized load, and time-correlated load) are realizable in a stochastic analysis.

Another significant finding in reference 6 was the relative computational costs of performing analyses using the MFB and SSB methods. For linear models, the MFB method is much more efficient than the SSB method. For nonlinear models, the MFB multi-dimensional search is much more expensive that the SSB method, while the MFB one-dimensional search requires less time that the SSB method.

Since the MFB multi-dimensional search method as now implemented is prohibitively expensive, the options for methods that can practically be applied to nonlinear systems are the MFB one-dimensional search and the SSB methods.

Based on the results in reference 6, the one-dimensional search is able to predict the maximized loads for nonlinear systems. In addition, it requires less computer resources than the SSB method. These factors show the utility of using the MFB one-dimensional search as a means of obtaining time-correlated gust loads for aircraft with nonlinear control systems.

REVIEW OF THE MFB METHOD FOR LINEAR SYSTEMS

The purpose of this section of the manual is to review the basic matched filter concepts. A detailed theoretical development of the MFB linear method can be found in reference 2. The signal flow diagram in figure 1 outlines the implementation and illustrates the intermediate and final products of the method.

Transfer-function representations of atmospheric turbulence and airplane loads are combined in series and represent the "known dynamics" boxes in the figure. A transfer-function representation of the von Karman spectrum in chosen for the gust filter. Load y is the load to be maximized. Loads z_1 through z_n are the loads to be time correlated with load y. There are three major steps in the process:

Step i — The application of an impulse function of unit strength to the combined linear system, producing the impulse response of load y, h(t).

Step ii — The normalization of this impulse response by the square root of its energy and then reversing it in time.

Step iii — The application of this normalized reversed signal to the combined linear system, producing time histories of load y and time histories of loads z_1 through z_n.

For simplicity of discussion these three steps will be referred to as the "MFB linear method."

The square root of the energy is defined by

$$\sqrt{energy} = \sqrt{\int_0^{t_o} h^2(t)\,dt} \qquad (1)$$

where h(t) is the impulse response of load y. For a stable system the numerical value of the quantity on the right side of equation (1) approaches a constant value as t_o is increased.

The selection of t_o is based on the time required for the load impulse responses to damp out to near zero. Figure 1 shows that the impulse response of load y has essentially damped out at time t_o. Thus, t_o is large enough for the response shown in figure 1. Too large a t_o value will, however, unduly increase the amount of computations required. The analyst must choose a value that provides accurate results while minimizing the use of computer resources.

Within the time history of load y in step iii, the maximum value is y_{max}. For linear systems, the theory guarantees that no other signal similarly normalized will produce a value of y larger than y_{max}. This guarantee is a fundamental result of the MFB linear method.

MFB ONE-DIMENSIONAL SEARCH DESCRIPTION

The goal of Matched Filter Theory as applied to nonlinear systems is the same as that for linear systems: to find the maximized response time history, the maximum value of the response within that time history, and the time-correlated response time histories. Because the systems are not linear, the superposition principle of the MFB linear method no longer holds and the solutions for maximized loads cannot conveniently be obtained directly. The only practical means of finding the excitation waveform that maximizes y_{max} is a search procedure. The search is conducted systematically, subject to the constraint that the excitation waveform have a "unit" energy.

Because superposition no longer holds, the magnitude and character of the responses are not necessarily proportional to the magnitude of the input. For the remainder of this paper two input magnitudes are important: k, the strength of the initial impulse; and σ_g, the design value of the gust intensity. (For the MFB linear method, the magnitude of both of these quantities was unity.) For nonlinear systems and a specific σ_g, the shape of the excitation waveform is a function of k , and, consequently, the quantity y_{max} is also a function of this parameter.

The one-dimensional search procedure performs a systematic variation of the quantity k to find the shape of an excitation waveform that maximizes y_{max}. Figure 2 contains a signal flow diagram for this search procedure. Figure 2 is very similar to figure 1, but contains some subtle yet important differences that are indicated by the shaded boxes and by quotation marks. In figure 2 the initial impulse has a non-unity strength; the aircraft loads portion of the known dynamics box contains nonlinearities; and the shape of the excitation waveform and the value of y_{max} are functions of the initial impulse strength. In addition, the "matched" excitation waveform and the "matched" load are shown in quotes because, for nonlinear systems, there is no guarantee that y_{max} is a global maximum.

The application of the one-dimensional search procedure is as follows:

Step 1 Select a specific design value for σ_g.

Step 2 Select a set of values for k.

Step 3 For each value of k, perform steps i through iii of the MFB linear method to obtain a set of "y_{max}" values and also the corresponding "matched" excitation waveforms.

Step 4 From step 3 above, find the maximum value of y_{max} and its corresponding "matched" excitation waveform.

As seen in the first pass (upper half) of figure 2, the variation of the impulse strength (k) affects the MFB one-dimensional search analysis by changing the shape of the excitation waveform. For sufficiently low impulse strengths, the shape of the excitation waveform for nonlinear models will be invariant with k. While in this invariant region, the excitation waveforms will be similar to those that are obtained for linear models. For larger intensities the system nonlinearities are engaged and will cause the impulse responses and corresponding excitation waveforms to change shape. Consequently, they will differ from those obtained by linear systems.

As seen in the second pass (lower half) of figure 2, the gust intensity affects the MFB one-dimensional search analysis by scaling the excitation waveform prior to being applied to the nonlinear model. Consequently, a low gust intensity should result in the nonlinear model behaving linearly. As gust intensity is increased beyond some threshold the nonlinear model response will begin to deviate from that of its linear counterpart.

The effects of varying impulse strength and gust intensity will be discussed further in the numerical results section of the paper.

SELECTION OF GUST INTENSITIES

The purpose of this section of the paper is to present the reasoning behind the selection of the values of σ_g. Reference 6 describes the selection of gust intensities for the MFB and SSB methods.

The following equation, from reference 7, expresses the "design value" of quantity y as defined in the design envelope criterion

$$y_{design} = \bar{A}_y U_\sigma \qquad (2)$$

where the quantity $\bar{A}_y$ is the RMS value of quantity y per unit RMS gust intensity, obtained from a conventional random process analysis of the airplane and U_σ is specified in the criterion. From reference 8 the quantity U_σ in equation (2) is shown to be the product of the gust RMS value and the design ratio of peak value of load to RMS value of load, and, therefore the quantity y_{design} is interpreted as a peak value.

Reference 4 shows that, as a consequence of the normalization of the excitation waveform by the square root or its own energy and the use of unity gust intensity, the quantity y_{max} from the MFB linear method is equal to the quantity $\bar{A}_y$ from a conventional random process analysis, or

$$y_{max}(\sigma_g = 1) = \bar{A}_y \tag{3}$$

In equation (3) y_{max} is interpreted as an RMS value, not a peak value. Substituting equation (3) into equation (2), y_{design} is now

$$y_{design} = y_{max}(\sigma_g = 1)U_\sigma \tag{4}$$

If, in performing the MFB linear method, U_σ is used for the gust intensity then the quantity y_{max} is equal to

$$y_{max}(\sigma_g = U_\sigma) = \bar{A}_y U_\sigma \tag{5}$$

The right hand sides of equations (2) and (5) are seen to be equal, therefore

$$y_{design} = y_{max}(\sigma_g = U_\sigma) \tag{6}$$

Two options for the value of σ_g have been offered: $\sigma_g = 1$, for which y_{design} is defined by equation (4); and $\sigma_g = U_\sigma$, for which y_{design} is defined by equation (6). When analyzing a linear system the choice of σ_g is irrelevant because the same value of y_{design} will be obtained in either case. However, when nonlinearities are introduced into aircraft control systems, loads are not simply proportional to gust intensity. Consequently, σ_g should be set to U_σ in the MFB nonlinear calculations, or

$$\sigma_{g\,MFB} = U_\sigma \tag{7}$$

and the resulting "y_{max}" values from the method should be interpreted as y_{design}.

DESCRIPTION OF MFB1DS

MFB1DS is a FORTRAN 77 program which performs the Matched-Filter-Based one-dimensional search. The MFB1DS solution procedure follows what was outlined in figure 2. The code must be run once for each combination of gust intensity and output quantity for which a maximized value is desired.

Figure 3 shows the solution procedure used by MFB1DS. The program first reads the input data then generates the impulse responses for each k value by calling the simulation subroutine. The simulation subroutine uses a public domain ordinary differential equation solver to generate output time histories using a user defined subroutine containing the aircraft equations of motion. Next, the excitation waveforms are generated by reversing the impulse responses in time and normalizing them by the square root their respective energies. Then, the simulation subroutine is again called for each of the normalized excitation waveforms to obtain the "maximized" load response time histories. Finally, the pertinent output quantities are written to computer files.

This section of the manual describes the main parts of MFB1DS.

Required Files

Six files are required to run MFB1DS.

MFB1DS.F	Main program, see Appendix A
MFB1DS.INC	File containing the common blocks used by MFB1DS.F, see Appendix B
MODEL.F	User supplied file containing the subroutine EQSMOT, the aircraft equations of motion, see Appendix C
MODEL.INP	User supplied file containing the input quantities required to run the code, see Appendix D
LSODE.F	Public domain ordinary differential equation solver
INTUTILS.F	Public domain subroutines used by LSODE.F

The two public domain files LSODE.F and INTUTILS.F are well documented in their respective source codes and will not be discussed in detail in this manual. These files were selected because of the fact that they are in the public domain. The user is encouraged to substitute more efficient ordinary differential equation solvers if available.

MFB1DS.F Subroutines

Subroutine DATAIN	Reads the input parameters from the input file
Subroutine SIMULATE	Computes output time histories using LSODE.F and MODEL.F
Subroutine SAVMATFORM	Saves output quantities to a MATRIX$_X$ readable file
Subroutine MATSAV	Used by Subroutine SAVMATFORM to save data in MATRIX$_X$ format
Subroutine SAVASCIIFORM	Saves output quantities in a more easily readable ascii file

MODEL.F

MODEL.F is a user supplied file containing the subroutine EQSMOT, the equations for the gust filter in series with the aircraft equations of motion. This subroutine uses variables x (vector of states) and u (excitation) to obtain xd (derivative of x) and y (vector of outputs).

The following lines of code must be present at the top of the file MODEL.F:

```
      subroutine EQSMOT(neq,t,x,xd)
      parameter(maxout=20)
      double precision x(*), xd(*), y(maxout)
      double precision t, tstart, tend, u0, u1
      common /eqsmotcom/y,tstart,tend,u0,u1
c
      u=(t-tstart)*(u1-u0)/(tend-tstart)+u0
```

In spite of the fact that the user specifies a desired time step, the ordinary differential equation solver LSODE calls the EQSMOT subroutine at values of time (t) between time steps. Consequently, the equation on the last line of the above code is included to provide a linear interpolation for values of u between time steps.

A linear system can be used to demonstrate how x, xd, y and u are related. For a linear system only, the equations of motion can be written in state space form:

$$\{xd\} = [A]\{x\} + [B]u$$
$$\{y\} = [C]\{x\} + [D]u \qquad (8)$$

While equation (8) is linear, the relationship between x, u and xd is, in general, a nonlinear one.

Description Of Impulse Input

Since the impulse generating procedure is one of the key components of the program and there are several methods that can be used to generate impulse functions, a few words describing the impulse function procedure implemented in the program are warranted. The procedure chosen to generate the impulse function in this code is straightforeword and can be found in main program listing in Appendix A. In general, the impulse input function, as seen by the differential equation solver, is a ramp up from zero at the first time step to a value of k/(2*deltat) at the second time step, a constant value of k/(2*deltat) between time steps two and three, and a ramp down to zero between time steps three and four.

Description Of Output

Key information is written to standard out (unit 6) as shown in Appendix E. This information allows the user to monitor the progress of the program. Two additional output files can be created which contain time history and correlated output information. Both of these output files are ascii files: one is in a MATRIX$_X$ readable form and the other is in a more easily read form. The variables contained in the MATRIX$_X$ readable file are discussed below. The quantities found in the other output file are self explanatory and will not be discussed here.

The following scalar quantities are written to the MATRIX$_X$ file:

sigmag	Gust intensity.
noutmx	The output quantity to be maximized.

deltat — The time step.

tmaximp — The length of the impulse responses.

The following arrays can also written to the MATRIXχ file:

allkvals — This vector has length nkvals, and stores all the k values used in the analysis.

maxout — The maximum output values for each k value are stored in each column. Thus, this array will be an nout by nkvals array.

impres# — An array of this form is created for each output quantity. For instance, for output 1 an array named "impres1" will be created and for output 5 an array named "impres5" will be created. Each column of these arrays are impulse response time histories for one of the k values. Thus, these are ntsteps by nkvalues arrays.

wavef# — An array of this form is created for the output to be maximized. For instance, if output 6 is to be maximized, then an array named "wavef6" will be created. Each column of this array is an excitation waveform matched to output # for a k value. Thus, this is an ntsteps by nkvalues array.

exresp# — An array of this form is created for each output quantity. For instance, for output 1 an array named "exresp1" will be created. Each column of this array is an excitation waveform response for a k value. Thus, these are (2*ntsteps+1) by nkvalues arrays.

NUMERICAL EXAMPLE

This section describes the step by step process of obtaining numerical results at a gust intensity of 1,530 in/sec.

Step 1 - Create a subroutine containing the gust filter in series with the aircraft equations of motion.

The nonlinear simulation model of the ARW-2 drone aircraft equipped with a nonlinear control system was used in this example. This model was connected in series with a transfer-function representation of atmospheric turbulence. The transfer function used in this example was (ref. 8)

$$\frac{w_g}{\eta} = \sigma_g \sqrt{\frac{L}{\pi V}} \frac{[1+2.618(L/V)s][1+0.1298(L/V)s]}{[1+2.083(L/V)s][1+0.823(L/V)s][1+0.0898(L/V)s]} \qquad (9)$$

which approximates the square root of the von Karmon power spectral density function. The quantity σ_g is the intensity of the gust or standard deviation -- which, assuming zero mean, is also equal to the root-mean-square, or RMS, value -- of gust velocity.

Figure 4 shows a block diagram of the ARW-2 simulation model and includes the aeroelastic plant, a gust load alleviation (GLA) control law, and nonlinear control elements. The aeroelastic plant is a linear, s-plane aeroelastic half-model consisting of

two longitudinal rigid-body modes and three symmetric flexible modes. Unsteady aerodynamics were obtained using the doublet lattice method (ref. 9). The model also includes the dynamics of the control surface actuators. The two-input/two-output GLA control law was obtained using a Linear Quadratic Gaussian design approach with the intent of reducing wing root bending moment (ref. 10). The nonlinear elements impose deflection limits of ±1" on the elevators and 0" to +1" on the ailerons to simulate spoilers. The model contains 32 states, and the analysis conditions are at a Mach number of 0.86 and an altitude of 24,000 feet.

The resulting combined system has a single input (white noise) and many outputs including the gust velocity (y(14)). The output quantities for this model are as follows:

y(1)-y(3)	= No physical interpretation
y(4)	= Tip acceleration
y(5)	= Fuselage acceleration
y(6)	= Wing root bending moment (WRBM)
y(7)	= Wing root shear (WRS)
y(8)	= Wing outboard bending moment (WOBBM)
y(9)	= Wing outboard torsion moment (WOBTM)
y(10)	= Elevator deflection
y(11)	= Elevator rate
y(12)	= Aileron deflection
y(13)	= Aileron rate
y(14)	= Gust velocity

Appendix C contains the FORTRAN version of the analytical model. This model was created using MATRIX$_X$ SYSTEM BUILD (ref. 11) and converted to FORTRAN using the HYPERCODE (ref. 12) package. As shown in Appendix C, the appropriate lines of code were added to the model created by the HYPERCODE package to make it compatible with the differential equation solving scheme used in MFB1DS.

Step 2 - Create an input file for the model.

The length of the state vector and number of model output quantities are contained in the input file MODEL.INP and must be made compatible with the analytical model created in step 1. Other input quantities like the gust intensity, identifying the output quantity to be maximized, and the range of k values must also be selected. (Trial and error will be required to find the range of k values where the maximum load value is obtained.) In addition to these parameters, the length of time for the impulse responses and the time step must be chosen. The length of the impulse responses should be only long enough to allow the output quantity to damp out to a relatively small value as shown in the example in figure 4 (a), while the time step should be chosen in accordance with the model response characteristics.

The input file used in the ARW-2 analysis is shown in Appendix D.

Step 3 - Modify the parameters.

Appendix B contains a listing of MFB1DS.INC and a description of the parameters and variables contained there. The parameters found in the file MFB1DS.INC and MODEL.F must be made large enough to accommodate the analytical model (step 1) and the input file (step 2). Computer storage can be minimized by making parameters "maxstates" and "maxout" equal to the minimum values required for a given analytical model.

Step 4 - Compile the code.

The following statement compiles the code on a SUN workstation.

```
f77 MFB1DS.F MODEL.F LSODE.F INTUTILS.F
```

Step 5 - Execute the code.

Appendix D shows the initial input file used in the analysis of the ARW-2 model. The initial value of gust intensity was 1.530 in/sec (1.5X85 ft/sec). This initial value of gust intensity was chosen so that the nonlinearities would be invoked in this example. Nine k values were used ranging from 10 to 15,000. The output quantity to be maximized was y(6), wing root bending moment, WRBM. The standard output from this run is shown in Appendix E.

The following statement will run the code on a UNIX system

```
a.out < MODEL.INP
```

Step 6 - Examine output.

Of the nine k values used in the analysis, all the intermediate results for three of the nine k values are shown in figure 5. Beginning with plot (a), the impulse responses for the three k values are shown. Plot (b) gives the corresponding excitation waveforms obtained from these impulse responses. Plot (c) shows the critical gust profiles and plot (d) shows the maximized load responses created by the gust input. Plot (e) depicts maximum WRBM as a function of initial impulse strength: the circles are actual numerical results; the solid line is a faired line. For this particular example, a value of k of about 2,410 creates an excitation waveform and critical gust profile that yields the largest maximized value of WRBM, 296,994 in-lbs.

Step 7 - Vary parameters.

The user may wish to refine the results shown in figure 5. With this in mind, a new range of k values was selected and the case was rerun. Figure 6 shows the results of using the new set of k values distributed between 400 and 6,000 producing a much smoother curve. The maximum load value obtained using the refined k distribution was 296,804 in-lbs at a k value of approximately 2,173. For this particular airplane and set of flight conditions a larger load value was not obtained with the new range of k values.

EXAMINATION OF RESULTS

This section has been included in the manual to provide the user with insight into how the results from MFB1DS should be interpreted. The results presented in the previous section will be shown along with results obtained using two additional gust intensities. To obtain results at different gust intensities, the sigma value of the input file shown in Appendix D was changed and MFB1DS was rerun. In addition, answers were obtained for each of the three gust intensities using a linearized version of the model. For comparison these linear answers will be plotted with the results from the nonlinear model. Note that when using a linear model, only one k value needs to be used because

the answer will not be a function of k. Consequently, the maximum load value plotted versus k is a horizontal line for a linear model.

The results for gust intensity values of 1,020 in/sec, 1,530 in/sec and 2,040 in/sec are shown in figure 7. Before proceeding, it is important to point out that results are problem dependent, and while these results are typical of all the aircraft examined by NASA, different aircraft with different nonlinearities may exhibit different trends. With this in mind, two important points will be made concerning the general trends exhibited in figure 7. First, regardless of the gust intensity value, the maximum load is constant for small values of k. Consequently, when searching for the maximum load, the range of interest for the k values can be limited on the low end. Second, the maximum load value decreases toward some relatively small value as k is made very large. Thus, the range of interest for k values can also be limited on the high end. What happens in between these two extremes depends on the specific gust intensity.

In figure 7 the maximum load values are constant for k less that 400. In plot 7(a) a significantly larger load value was not found when k was increased beyond 400, while for plots 7(b) and 7(c) a larger value was found. This indicates that the character of the results (i.e., the shape of the maximum load versus k plot) and the specific k value that produces the maximum load value are functions of gust intensity. But, the range of k values where the maximum will be found is generally limited, and in this case the range is roughly between 400 and 10,000. It should also be noted that the difference between answers from linear and nonlinear models is also a function of the gust intensity. Larger gust intensity values generally result in larger differences between answers from linear and nonlinear models. Here, this difference is 2% for the lowest gust intensity and 18% for the largest gust intensity.

CONCLUDING REMARKS

This manual has reviewed the theory behind the Matched-Filter-Based one-dimensional search procedure. The code that performs this procedure has been discussed and example numerical results were presented and interpreted. The code, MFB1DS, is available in a self contained form. It has all the required equation solvers and files necessary to run the example problem. The user is, however, encouraged to modify the existing code by inserting more efficient routines if available.

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12. HYPER_BUILD 7.0 User's Guide: Integrated Systems Inc. October, 1988.

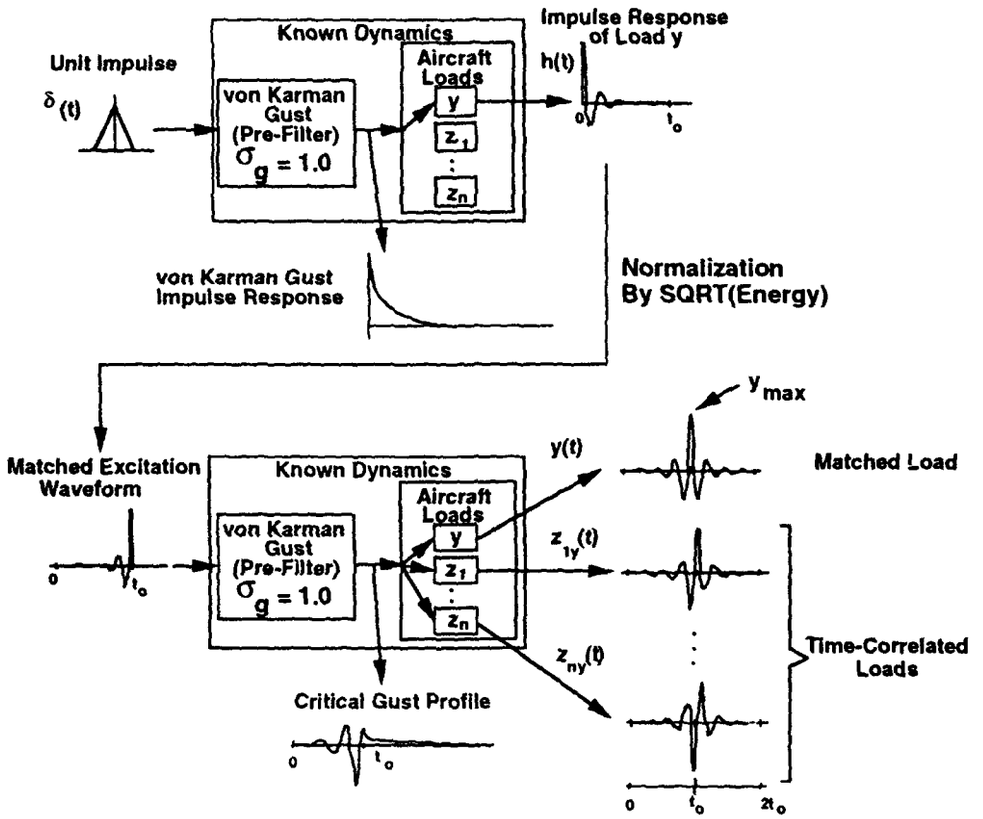


Figure 1. MFB linear method signal flow diagram.

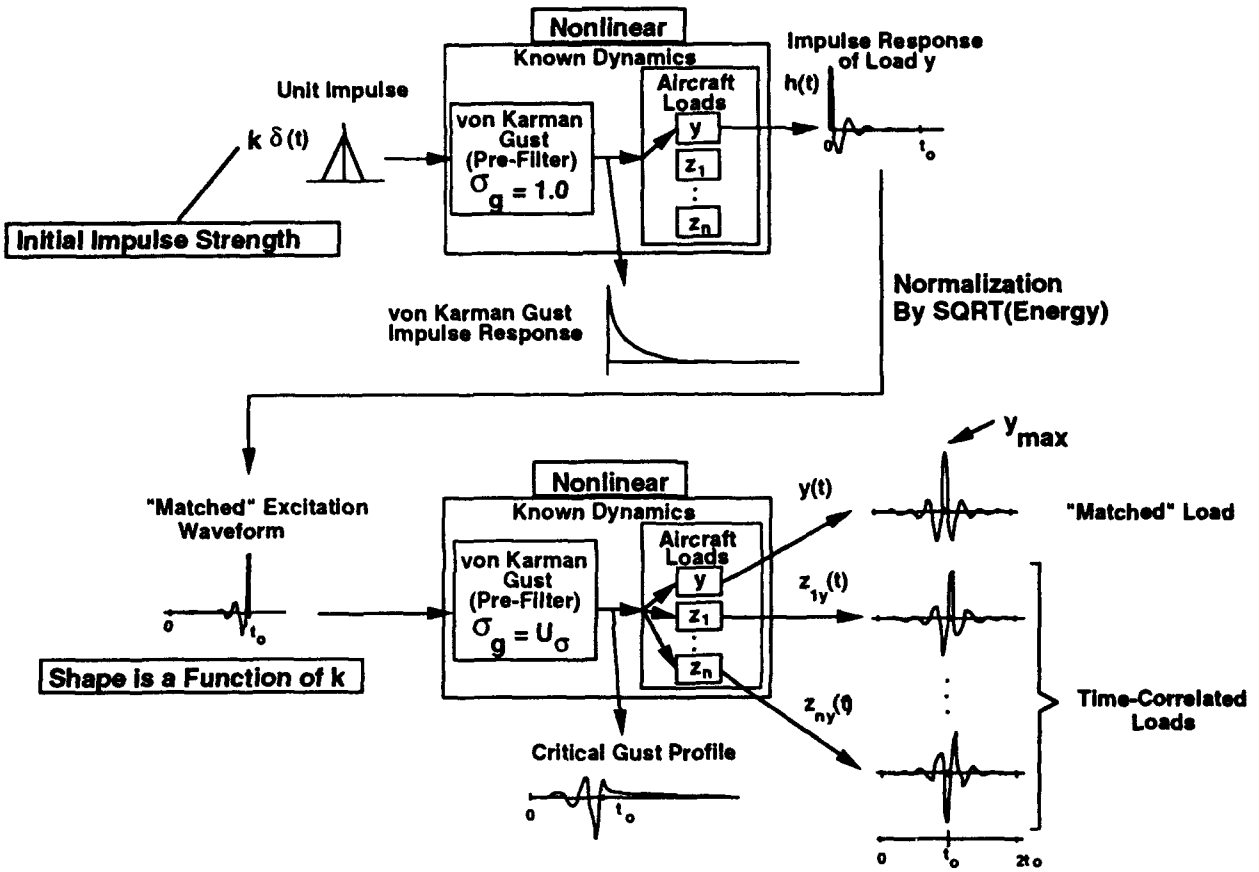


Figure 2. Nonlinear MFB signal flow diagram for the one-dimensional search.

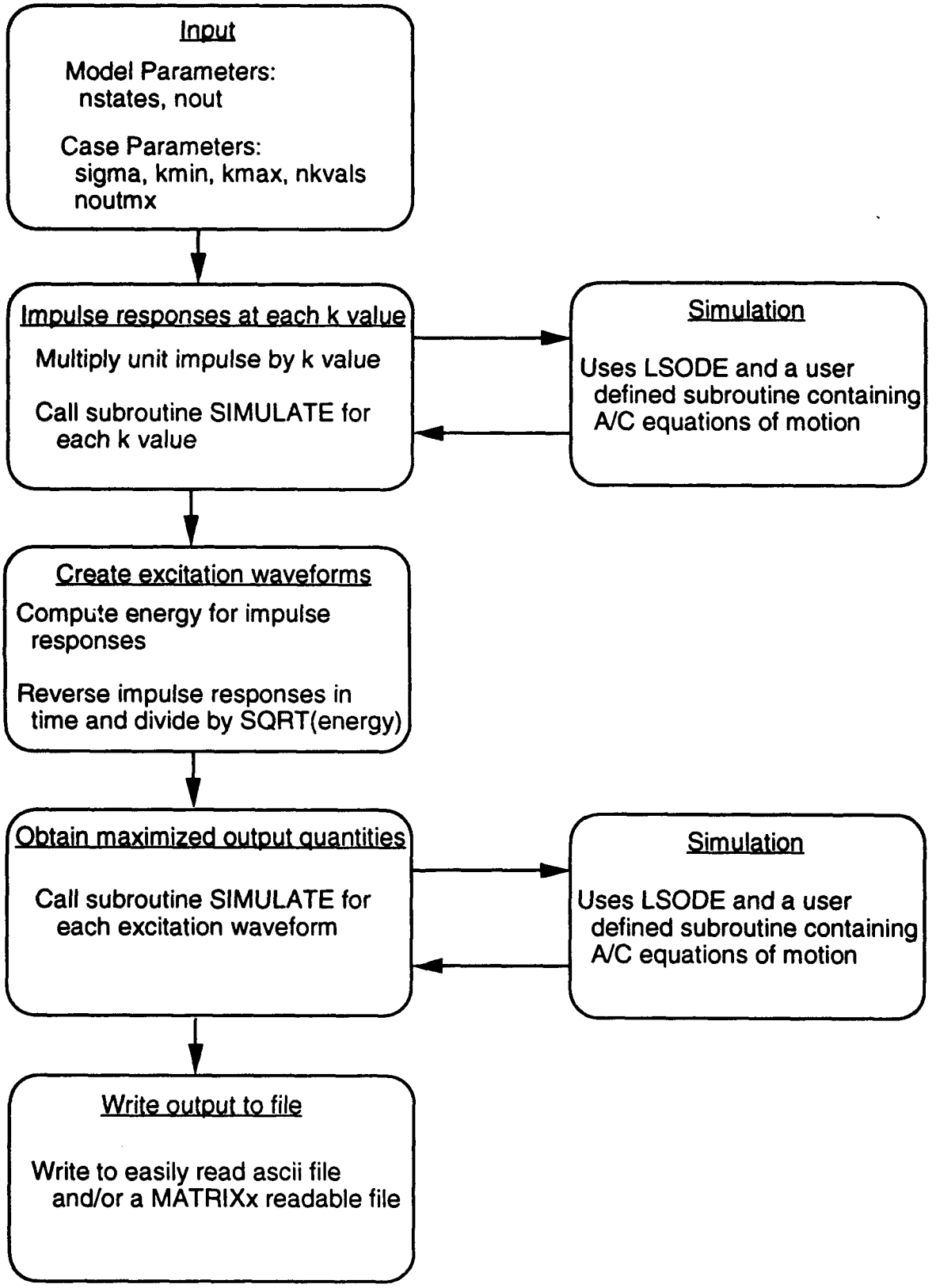


Figure 3. MFB1DS solution procedure.

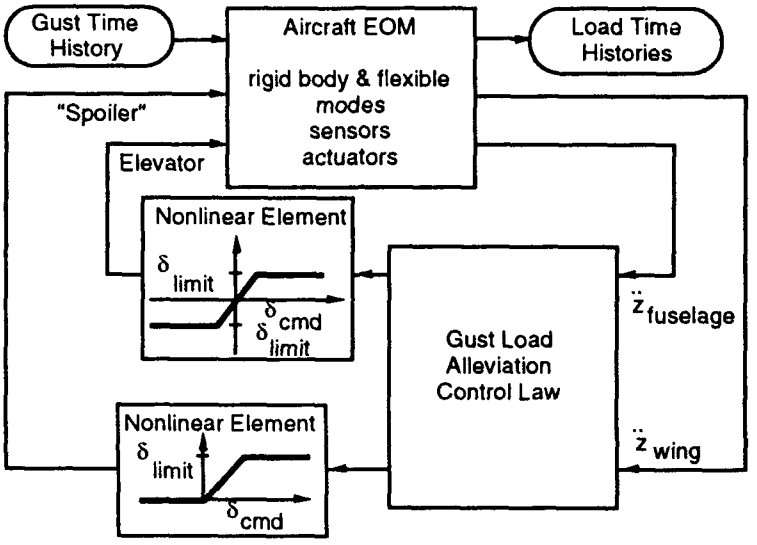
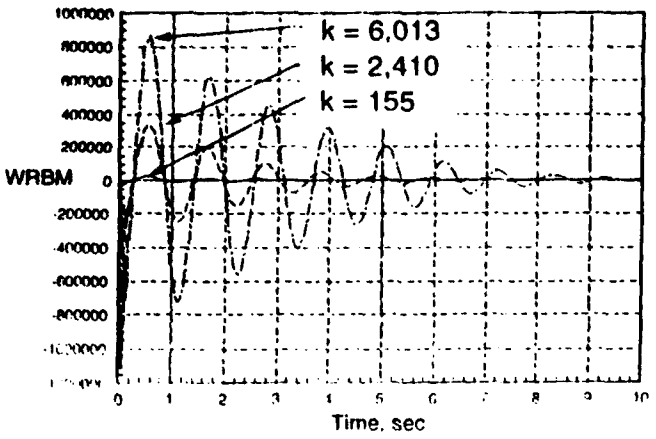
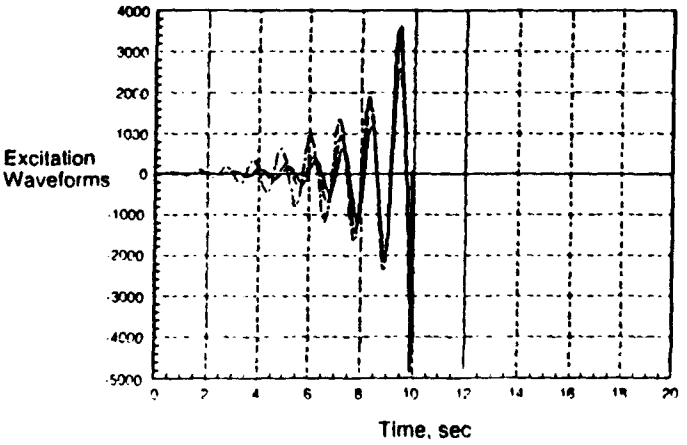


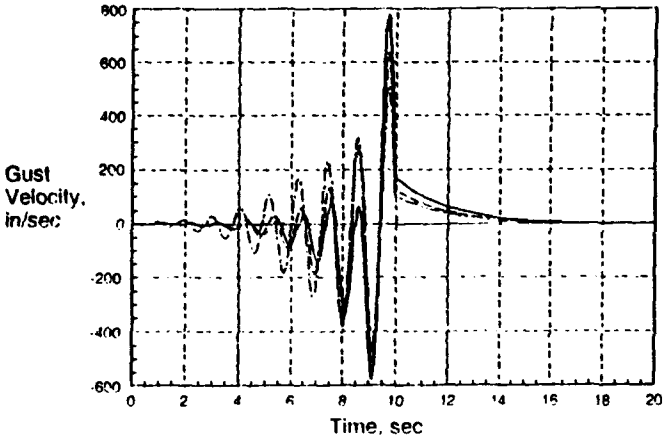
Figure 4. Nonlinear block diagram for the ARW-2 drone aircraft.



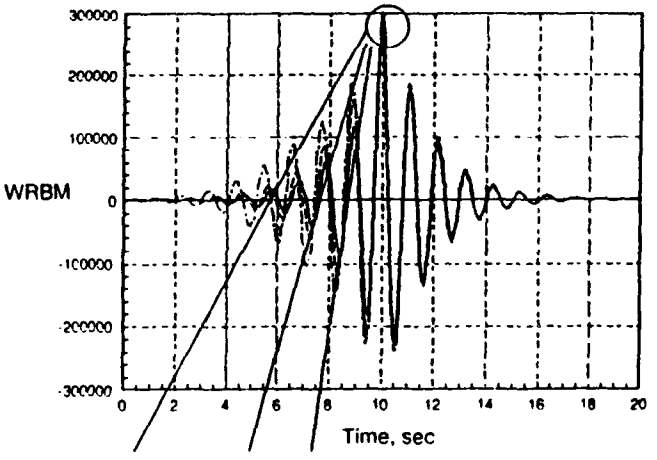
(a) Impulse responses.



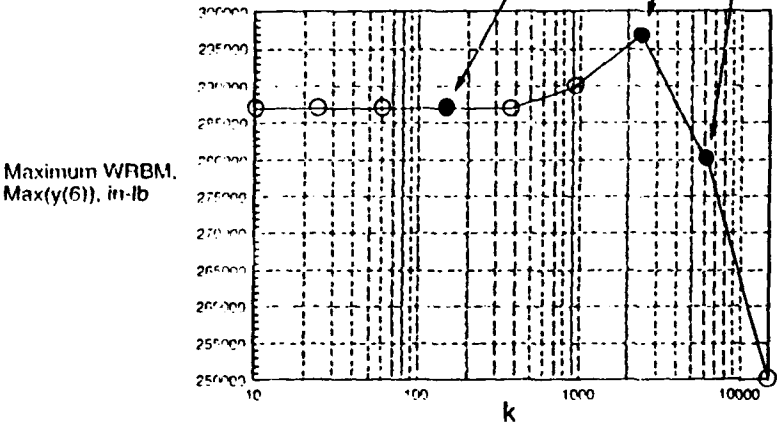
(b) Excitation waveforms (WRBM time histories that have been normalized, reversed and multiplied by σ_g).



(c) Critical gust profiles.



(d) Wing root bending moment time histories.



(e) Maximum wing root bending moment.

Figure 5.- Example calculations illustrating the effects of k variation on maximum WRBM, σ_g = 1.530 in/sec.

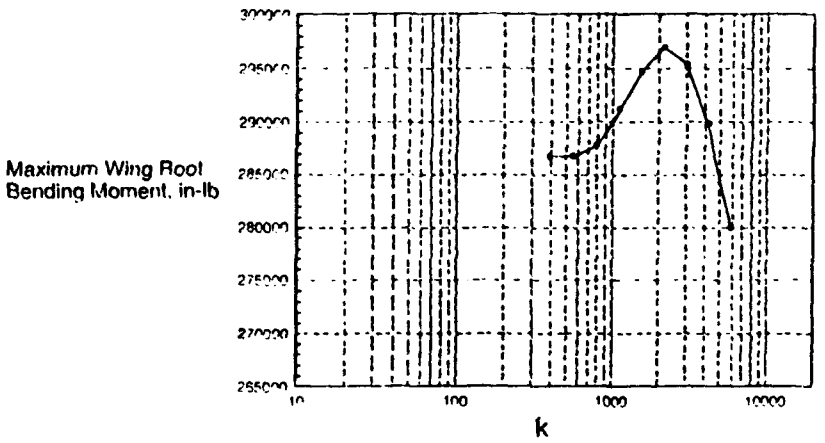
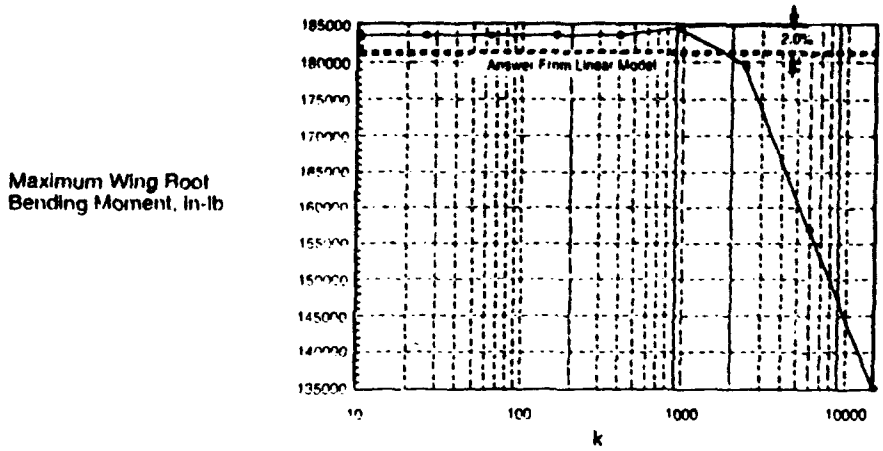
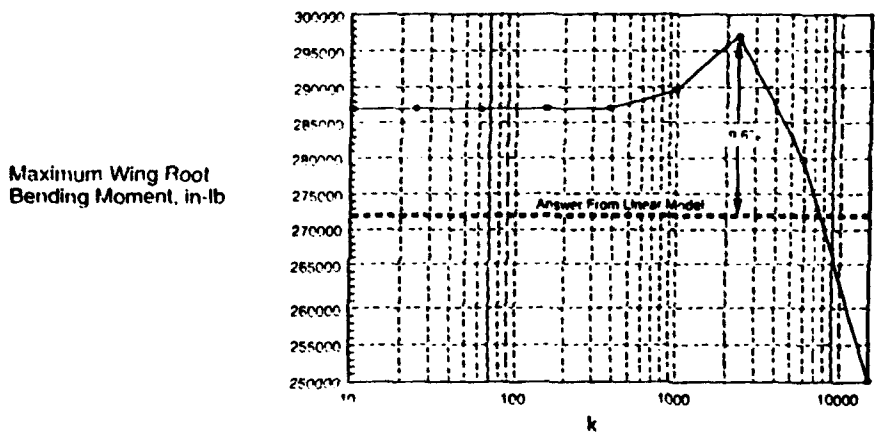


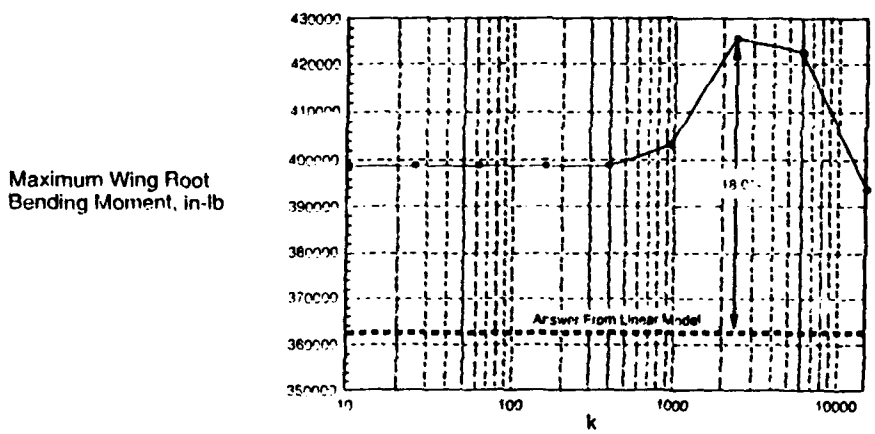
Figure 6.- Maximum WRBM versus k using a refined range of k values, σ_g= 1.530 in/sec.



(a) σ_g = 1,020 in/sec (85 ft/sec)



(b) σ_g = 1,530 in/sec (1.5X85 ft/sec)



(c) σ_g = 2,040 in/sec (2X85 ft/sec)

Figure 7.- Maximum WRBM versus k for 3 gust intensities.

Appendix A - MFB1DS.F source code listing

```
      program MFB1DS
c*************************************************************
c*                                                           *
c*   MFB1DS.F:       a matched-filter-based method of obtaining *
c*                   maximized and time-correlated gust loads for *
c*                   linear and nonlinear aircraft           *
c*                                                           *
c*   written by:     Robert C. Scott                         *
c*                   NASA Langley Research Center MS/340     *
c*                   Hampton, VA 23665                       *
c*                                                           *
c*                   804-864-2838                            *
c*                                                           *
c*   compile:        f77 MFB1DS.F MODEL.F LSODE.F INTUTILS.F *
c*                                                           *
c*                   where MODEL.F is the a/c model          *
c*************************************************************
      include 'MFB1DS.INC'
      character*80 ctitle
      character*80 ccase
      common /case/ccase
      common /title/ctitle
      dimension yth(maxout,2*maxtsteps)

c  **read input parameters
      call DATAIN

      write(6,*) ctitle
      write(6,*) ccase

c  **calculate number of time steps for impulse response
      ntsteps=int(tmaximp/deltat + 0.001) + 1
      write(6,*) ' '
      write(6,*) '  length of simulation for impulse responses = ',
     +          tmaximp
      write(6,*) '  number of time steps for impulse responses = ',
     +          ntsteps
      write(6,*) ' '
      write(6,*) '  length of simulation for excitation responses = ',
     +        2*tmaximp
      write(6,*) '  number of time steps for excitation responses = ',
     +        2*ntsteps-1
      write(6,*) ' '
      write(6,*) '  gust intensity = ', sigma
      write(6,*) ' '

c  **create array of zeros for inpulse input
      do 100 i=1,(2*ntsteps-1)
  100   aimpt(i)=0.0

c  **calculate k values
      do 200 k=1,nkvals
```

```
      if (nkvals.eq.1) then
        allkvals(1)=akmin
      else
        allkvals(k)=10.0**( (k-1)*(log10(akmax)-log10(akmin))/
     +                 float(nkvals-1)+log10(akmin) )
      endif
  200 continue

c **obtain impulse response for each k value
      write(6,*) 'calculating impulse responses for each k value'
      do 300 k=1,nkvals
        ak=allkvals(k)
c **assign impulse strength k to array aimpt
        aimpt(2)=ak/deltat/2
        aimpt(3)=ak/deltat/2
c **call subroutine SIMULATE to obtain impulse response
        call SIMULATE(yth,aimpt,ntsteps)
        do 300 iout=1,nout
        do 300 j=1,ntsteps
  300     aimp_res(j,iout,k)=yth(iout,j)

c **calculate normalized excitation waveforms
      write(6,*) 'calculating normalized excitation waveforms'
      do 400 k=1,nkvals
        i=noutmx
        energy=0.0
        energy=aimp_res(1,i,k)**2 + aimp_res(ntsteps,i,k)**2
        do 401 j=2,ntsteps-1
  401     energy = energy+2*(aimp_res(j,i,k))**2
        energy = sqrt( tmaximp*energy*.5/float(ntsteps)/3.14159 )
        write(6,*) ' ', k, '  k = ', allkvals(k),
     +     '  sqrt(energy) of output', noutmx, ' = ', energy

        do 402 j=ntsteps,1,-1
  402     wave(ntsteps-j+1,k)=sigma*aimp_res(j,i,k)/energy

  400 continue

c **obtain maximized responses
c   Note that here the length of the simulation is 2*tmaximp and the
c   number of time steps for maximized responses is 2*ntsteps-1

      write(6,*) 'calculating maximized responses'
      do 500 k=1,nkvals

        do 501 j=1,ntsteps
  501     aimpt(j)=wave(j,k)
        do 502 j=(ntsteps+1), (2*ntsteps-1)
  502     aimpt(j)=0.0
        call SIMULATE(yth,aimpt, (2*ntsteps-1) )
        do 503 j=1,(2*ntsteps-1)
        do 503 i=1,nout
  503     aexc_res(j,i,k)=yth(i,j)
```

```
      write(6,*) ' ', k,' k = ',allkvals(k),
     +         ' maximum value of output ',
     +         noutmx, ' = ', aexc_res(ntsteps,noutmx,k)

  500 continue

c **save data
      write(6,*) 'saving data'
      if (iouttype.eq.1.or.iouttype.eq.3) call SAVASCIIFORM
      if (iouttype.eq.2.or.iouttype.eq.3) call SAVMATFORM

      stop
      end

c-------------------------------------------------------------------------------
cl   SIMULATE: subroutine to obtain a time history for the a/c model l
c-------------------------------------------------------------------------------
      subroutine SIMULATE(yth,u,nsteps)
      include 'MFB1DS.INC'
      dimension yth(maxout,2*maxtsteps), u(2*maxtsteps)
      double precision y(maxout)
      double precision atol, rtol, xstate(maxstates)
      double precision rwork(22*10)*maxstates+(2*1+1)*maxstates)
      double precision tstart,tend, u0,u1
      integer neq,itol,itask,istate,iopt,lrw,liw,mf
      integer iwork(20+maxstates)
      common /eqsmotcom/y,tstart,tend,u0,u1

c **for guidance in selecting sizes for arrays rwork and iwork
c   see source code LSODE.F

      external EQSMOT

c **Setup input for LSODE
c   see LSODE.F for explanation of these parameters
c
      neq=nstates
c     tstart=starting time
c     tend=ending time
      itol=1
      rtol=1.0E-06
      atol=1.0E-10
      itask=1
      istate=1
      iopt=0
c     rwork=real work space
      lrw=22*10*maxstates+(2*1+1)*maxstates
c     iwork=imiginary work space
      liw=20+maxstates
c     jaceqs
      mf=23

      do 1 i=1,maxstates
    1   xstate(i)=0.0
```

```
      do 11 i=1,maxout
 11     yth(i,1)=0.0

      tstart=0.0
      tend=0
      do 2 i=2,nsteps
        tend=tend+deltat
          u0=u(i-1)
          u1=u(i)
        call LSODE(EQSMOT,neq,xstate,tstart,tend,itol,rtol,atol,
     +          itask,istate,iopt,rwork,lrw,iwork,liw,jaceqs,mf)
c  **If istate is not equal to 2 then an error has occurred
c    and the following will be printed
        if (istate.ne.2) write(6,*) 'istate = ', istate,
     +                      'tend = ', tend
        tstart=tend
        do 3 j=1,nout
 3        yth(j,i)=y(j)
 2    continue

      return
      end
c
c-------------------------------------------------------------------------
c|   DATAIN:  subroutine to read data from file MFB1DS.INP               |
c-------------------------------------------------------------------------
c
      subroutine DATAIN
      include 'MFB1DS.INC'
      character*80 ctitle
      character*80 ccase
      character*15 cdata, cmatrixx
      common /case/ccase
      common /title/ctitle
      common /fnames/cdata, cmatrixx
      character cdummy*80

c  **read model title
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt='(a80)') ctitle

c  **read case title
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt='(a80)') ccase

c  **read nstates, nout, noutmx
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt=*) nstates,nout, noutmx

c  **read tmaximp, deltat, nsubintvl
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt=*) tmaximp, deltat
```

```
c **read sigma, akmin, akmax, nkvals
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt=*) sigma, akmin, akmax, nkvals

c **read iouttype, impres, ixcres, iwave
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt=*) iouttype, impres, iexcres, iwave

c **read data file name
      read(unit=5,fmt='(a80)') cdata
      read(unit=5,fmt='(a12)') cdata

c **read matrixx file name
      read(unit=5,fmt='(a80)') cdummy
      read(unit=5,fmt='(a12)') cmatrixx

      close(unit=5)

      return
      end

c-------------------------------------------------------------------------------
cl   SAVMATFORM:  subroutine to write output to a MATRIXx readable rile  l
c-------------------------------------------------------------------------------
      subroutine SAVMATFORM
      include 'MFB1DS.INC'
      dimension rtemp(2*maxtsteps,maxstates)
      character*10 varname, cnout
      character*15 cdata, cmatrixx
      common /fnames/cdata, cmatrixx

      open(unit=1,file=cmatrixx)
      rewind(1)

c **write gust intensity
      rtemp(1,1)=sigma
      call MATSAV(1,'sigmag', 2*maxtsteps, 1,
     +          1, 0, rtemp, rtemp, '(1p2e24.15)')

c **write k values for each case
      do 1 i=1,nkvals
 1      rtemp(i,1) = allkvals(i)
      call MATSAV(1,'kvals', 2*maxtsteps, nkvals,
     +          1, 0, rtemp, rtemp, '(1p2e24.15)')

c **write which output quantity was "maximized"
      rtemp(1,1)=noutmx
      call MATSAV(1,'noutmx', 2*maxtsteps, 1,
     +          1, 0, rtemp, rtemp, '(1p2e24.15)')

c **write time step
      rtemp(1,1)=deltat
      call MATSAV(1,'deltat', 2*maxtsteps, 1,
     +          1, 0, rtemp, rtemp, '(1p2e24.15)')
```

```
c **write tmaximp
      rtemp(1,1)=tmaximp
      call MATSAV(1,'tmaximp', 2*maxtsteps, 1,
     +          1, 0, rtemp, rtemp, '(1p2e24.15)')

c **write maximum output values
      do 3 i=1,nout
      do 3 k=1,nkvals
3       rtemp(k,i)=aexc_res(ntsteps,i,k)
      call MATSAV(1,'maxout', 2*maxtsteps, nkvals,
     +          nout, 0, rtemp, rtemp, '(1p2e24.15)')

c **write impulse responses
      if (impres.eq.1) then
        do 4 i=1,nout

          open(unit=50, status='SCRATCH')
          write(unit=50,fmt= '(i3)') i
          rewind(50)
          ndigits=1
          if (i.gt.9) ndigits=2
          if (i.gt.99) ndigits=3
          cnout='  '
          read(unit=50,fmt='(a3)') cnout(1:ndigits)
          close(50)

          varname='impres'//cnout
          do 5 k=1,nkvals
          do 5 nt=1,ntsteps
5           rtemp(nt,k)=aimp_res(nt,i,k)
          call MATSAV(1, varname, 2*maxtsteps, ntsteps,
     +          nkvals, 0, rtemp, rtemp, '(1p2e24.15)')
4       continue
      endif

c **write excitation responses
      if (iexcres.eq.1) then
        do 6 i=1,nout

          open(unit=50, status='SCRATCH')
          write(unit=50,fmt= '(i3)') i
          rewind(50)
          ndigits=1
          if (i.gt.9) ndigits=2
          if (i.gt.99) ndigits=3
          cnout='  '
          read(unit=50,fmt='(a3)') cnout(1:ndigits)
          close(50)
          varname='exresp'//cnout

          do 7 k=1,nkvals
          do 7 nt=1,(2*ntsteps-1)
7           rtemp(nt,k)=aexc_res(nt,i,k)
```

```
      call MATSAV(1, varname, 2*maxtsteps, (2*ntsteps-1),
     +         nkvals, 0, rtemp, rtemp, '(1p2e24.15)')
 6    continue
      endif

c  **write excitation waveforms
      if (iwave.eq.1) then

        open(unit=50, status='SCRATCH')
        write(unit=50,fmt= '(i3)') noutmx
        rewind(50)
        ndigits=1
        if (noutmx.gt.9) ndigits=2
        if (noutmx.gt.99) ndigits=3
        cnout='  '
        read(unit=50,fmt='(a3)') cnout(1:ndigits)
        close(50)

        varname='wavef'//cnout
        do 9 k=1,nkvals
        do 9 nt=1,ntsteps
 9        rtemp(nt,k)=wave(nt,k)
        call MATSAV(1, varname, 2*maxtsteps, ntsteps,
     +         nkvals, 0, rtemp, rtemp, '(1p2e24.15)')

      endif

      close(1)

      return
      end

c-------------------------------------------------------------------------------
c|   SAVASCIIFORM:  subroutine to write output to an scii file      |
c-------------------------------------------------------------------------------
      subroutine SAVASCIIFORM
      include 'MFB1DS.INC'
      character*80 ctitle
      character*80 ccase
      character*15 cdata, cmatrixx
      common /case/ccase
      common /title/ctitle
      common /fnames/cdata, cmatrixx

      open(unit=9, file=cdata)
      rewind(9)

c  **write general information
      write(9,'(a80)') ctitle
      write(9,'(a80)') ccase
      write(9,*) '    sigma = ', sigma
      write(9,*) '    matched output quantity = ', noutmx
      write(9,*) '    tmaximp  = ', tmaximp
      write(9,*) '    deltat  = ', deltat
```

```
c **write maximum output values for each case
      write(9,*) ' '
      write(9,*) '*************************************************'
      write(9,*) 'MAXIMIZED AND TIME CORRELATED MAX OUTPUT
QUANTITIES'
      write(9,*) '*************************************************'
      do 1 i=1,nout
        write(9,*) '  output quantity = ', i
        write(9,*) '  k value        maximum output value'
        do 1 k=1,nkvals
          write(9,*)  allkvals(k), '            ',
     +          aexc_res(ntsteps,i,k)
 1    continue

c **write impulse responses

      if (impres.eq.1) then
        do 2 k=1,nkvals
          write(9,*) ' '
          write(9,*) '*************************************************'
          write(9,*) 'IMPULSE RESPONSES'
          write(9,*) '*************************************************'
          write(9,*) ' kvalue = ', allkvals(k),
     +          ' maximized output quantity = ', noutmx
          do 2 iout=1,nout
            write(9,*) 'output quantity = ', iout
            do 2 j=1,ntsteps
              write(9,*) aimp_res(j,iout,k)
 2      continue
      endif

c **write excitation waveforms
      if (iwave.eq.1) then
        do 3 k=1,nkvals
          write(9,*) ' '
          write(9,*) '*************************************************'
          write(9,*) 'EXCITATION WAVEFORMS'
          write(9,*) '*************************************************'
          write(9,*) ' kvalue = ', allkvals(k),
     +          ' output quantity = ', noutmx
          do 3 j=1,ntsteps
            write(9,*) wave(j,k)
 3      continue
      endif

c **write excitation responses
      if (iexcres.eq.1) then
        do 4 k=1,nkvals
          write(9,*) ' '
          write(9,*) '*************************************************'
          write(9,*) 'EXCITATION WAVEFORM RESPONSES'
          write(9,*) '*************************************************'
          write(9,*) ' kvalue = ', allkvals(k),
```

```
     +            ' maximized output quantity = ', noutmx
          do 4 iout=1,nout
           write(9,*) 'output quantity = ', iout
           do 4 j=1,2*ntsteps-1
            write(9,*) aexc_res(j,iout,k)
4       continue
        endif


        close(9)

        return
        end

c-----------------------------------------------------------------------
cl   MATSAV: write variables to a file in matrixx format    l
c-----------------------------------------------------------------------
        subroutine MATSAV ( lunit, name, nr, m, n, img,
     +                  xreal, ximag, formt )
c
c ------------------------------------------------------------
c
c  MATSAV writes a matrix to a file in a format suitable for the
c       matrixx load operation.
c
c ---------------------------------------------------------------------------
c  param.  type              on input-                        on output-
c ---------------------------------------------------------------------------
c
c  lunit     integer     fortran logical unit number.        unchanged.
c
c  name character*(*)      name of the matrix.  one al-      unchanged.
c        (maximum          phabetic followed by up to 9
c         length 10)       alphanumeric characters.
c
c  nr        integer     row-dimension in the                unchanged.
c                        defining dimension or type
c                        statement in the calling
c                        program. nr must be greater
c                        than or equal to m.
c
c  m         integer     number of rows of the matrix        unchanged.
c
c  n         integer     number of columns of the            unchanged.
c                        matrix.
c
c  img       integer     if img = 0, the imaginary           unchanged.
c                        part (ximag) is assumed to
c                        be zero and is not saved.
c
c  xreal     double      real part of the matrix to          unchanged.
c            precision   be saved.
c
c  ximag     double      imaginary part of the matrix        unchanged.
```

```
c           precision    to be saved.
c
c  formt    character*(*)  string containing the for-      unchanged.
c           (maximum       tran format to be used for
c           length 20)     writing the elements of the
c                          matrix.
c
c -----------------------------------------------------------------
c
c  example:   the following fortran program generates an elementary
c             matrix in x and writes it to fortran unit 1.  assume
c             that unit 1 has been preallocated as file (data set)
c             test.
c
c
c            dimension x(20,3), dummy
c             do 200 j=1,3
c               do 100 i=1,10
c               x(i,j)=0.0d0
c         100   continue
c             x(j,j)=1.0d0
c         200  continue
c             call MATSAV( 1, 'amatrix', 20, 10, 3, 0,
c            $           x, dummy, '(1p2e24.15)' )
c             stop
c             end
c
c
c  after this program runs, invoke matrixx and type:
c
c          <> load 'test'
c
c  this will put x on the stack as stack-variable-name amatrix.
c
c -----------------------------------------------------------------
c
      integer         lunit, m, n, nr, img
      character*(*)     name, formt
      dimension        xreal(nr,1), ximag(nr,1)
      character       nam*10, form*20
c
c                      ---------------------------------------
c                       write header record.
c                      ---------------------------------------
      nam=name
      form=formt
      write(lunit,'(a10,3i5,a20)') nam,m,n,img,form
c                      ---------------------------------------
c                       write real-part of the matrix.
c                      ---------------------------------------
      write(lunit,form) ((xreal(i,j),i=1,m),j=1,n)
c                      ---------------------------------------
c                       write imaginary-part if nonzero.
c                      ---------------------------------------
```

```
if(img.ne.0) write(lunit,form) ((ximag(i,j),i=1,m),j=1,n)
return
end
```

Appendix B - MFB1DS.INC source code and description of parameters

```
      parameter(maxtsteps=5001, maxstates=40, maxout=20, maxkvals=20)
      common /modelinfo/nstates, nout
      common /siminfo/tmaximp, deltat, ntsteps, nsubintvl
      common /case_info/noutmx
      common /excitation_info/sigma, akmin, akmax, nkvals,
     +                  allkvals(maxkvals)
      common /t_hist/aimp_res(maxtsteps,maxout,maxkvals),
     +           aexc_res(2*maxtsteps,maxout,maxkvals),
     +              wave(maxtsteps,maxkvals),
     +             aimpt(2*maxtsteps)
      common /output/iouttype, impres, iexcres, iwave
```

The following is a description of the parameters:

maxtsteps	This parameter is the maximum number of time steps for the impulse responses. (maxtsteps ≥ tmaximp/deltat +1)
maxstates	This parameter is the maximum number of states in the equations of motion. (maxstates ≥ nstates)
maxout	This the maximum number of output quantities required by the aircraft model. (maxout ≥ nout)
maxkvals	This the maximum number of kvalues which can be run.

To change these parameters edit MFB1DS.INC. In addition, maxout is also found in MODEL.F and must be equal that the value in MFB1DS.INC. Computer storage can be minimized by making maxstates and maxout equal to the minimum values required for a given analytical model.

Appendix C - MODEL.F (ARW-2) source code listing

```
      subroutine EQSMOT(neq,t,x,xd)
      parameter(maxout=20)
      double precision x(*), xd(*), y(maxout)
      double precision t,tstart,tend, u0, u1
      common /eqsmotcom/y,tstart,tend,u0,u1
c
      u=(t-tstart)*(u1-u0)/(tend-tstart)+u0

c---------------------------------------- state-space system
c -- {nlarw2.controller.1}
c -- ss  c --
        y(1) = -5.244970395787696D-4*x(1) - 2.74700469140994399D-4*x(2)
     +        - 1.42946149773359D-3*x(3) + 5.56258182659891597D-6*x(4)
        y(2) = -3.287044785655141102D-3*x(1) - 6.743878267402364D-5*x(2)
     +        - 9.87258135859925306D-4*x(3) +
     +       9.9055669393222510 4D-6*x(4)
c---------------------------------------- 1 - u bounded limit
c -- {nlarw2.aileron limiter.2}
        y(3) = min( 0.0D0, max( -0.0174499999999999999D0, y(2) ) )
c       y(3) = min( 0.0174499999999999999D0, max(
c    +            -0.0174499999999999999D0, y(2) ) )
c---------------------------------------- state-space system
c -- {nlarw2.arw2 moD.3}
c -- ss  c --
        y(4) = -2846.473761074609D0*x(5) - 2699.57672001826D0*x(6) +
     +       6978.29411601534196D0*x(7) + 2.143502931550089 02D5*x(8) -
     +       22.3289763900712601D0*x(9) - 3.399062943714242D0*x(10) -
     +       9.13144768834393994D0*x(11) + 5.05173942454337099D0*x(12)
     +       + 75.7538109829206405D0*x(13) -
     +       0.958821750950846804D0*x(14) -
     +       0.8602462874967713 03D0*x(15) - 78.9585290740215004D0*x(16)
     +        + 43.922173428509 2698D0*x(17) +
     +       8.352989375082132 02D0*x(18) - 0.958821750950846804D0*x(19)
     +        - 0.8602462874967713 03D0*x(20)
        y(4) = y(4) - 78.9585290740215004D0*x(21) +
     +       43.922173428509 2698D0*x(22) + 8.352989375082132 02D0*x(23)
     +       - 1954.936245050993 99D0*x(24) -
     +       2.434167026240218 02D0*x(25) - 2597.51251185656298D0*x(26)
     +       + 37578.76249270933 02D0*x(27) -
     +       4.172084012088732 03D0*x(28) + 3.077920932009688 97D9*x(29)
     +       - 41.06334439731767 99D0*x(35) +
     +       3.573167209482252 97D-3*x(36)
        y(5) = -42.1666883238785903D0*x(5) + 16.36587051303695D0*x(6)
     +       - 453.129586109553202D0*x(7) + 351.460438213513001D0*x(8)
     +       - 0.599754637061799697D0*x(9) -
     +       4.632330831449649 02D-2*x(10) - 1.520116146415901D-2*x(11)
     +       - 6.863707281844977 01D-2*x(12) +
     +       0.483698505531560102D0*x(13) -
     +       0.969166155996266099D0*x(14) +
     +       0.519949869945893497D0*x(15) + 1.263028288468647 01D0*x(16)
     +        - 1.740007095918280 01D0*x(17) -
     +       1.189239579268164D-2*x(18) - 0.969166155996266099D0*x(19)
```

```
+      + 0.519949869945893497D0*x(20)
       y(5) = y(5) + 1.2630282884686470 1D0*x(21) -
+      1.7400070959182800 1D0*x(22) - 1.189239579268164D-2*x(23)
+      + 900.37846047579300 5D0*x(24) +
+      0.643799490067852503D0*x(25) + 1238.14470008882901D0*x(26)
+       + 291.107672370571898D0*x(27) -
+      9.35961201178221493D-2*x(28) + 3.81608823019032497D7*x(29)
+       - 0.652116307184986296D0*x(35) -
+      2.3240999259453940 1D-5*x(36)
       y(6) = 6384.77199999999698D0*x(6) - 26281.13000000000 1D0*x(7)
+      - 44252.8600000000997D0*x(8)
       y(7) = 81.7875899999999092D0*x(6) - 1061.036D0*x(7) -
+      2228.25D0*x(8)
       y(8) = -231.7430000000004D0*x(6) - 3091.725999999995D0*x(7) +
+      6162.581000000000604D0*x(8)
       y(9) = 28.7296599999999702D0*x(6) + 534.102200000001204D0*x(7)
+      - 5791.63599999999894D0*x(8)
       y(10) = x(24)
       y(11) = x(25)
       y(12) = x(27)
       y(13) = x(28)
       y(14) = x(35)
       y(15) = -2846.473761074609D0*x(5) - 2699.57672001826D0*x(6) +
+      6978.29411601534196D0*x(7) + 2.14350293155008902D5*x(8) -
+      22.328976390071260 1D0*x(9) - 3.399062943714242D0*x(10) -
+      9.131447688343939 94D0*x(11) + 5.05173942454337099D0*x(12)
+      + 75.7538109829206405D0*x(13) -
+      0.958821750950846804D0*x(14) -
+      0.860246287496771303D0*x(15) - 78.9585290740215004D0*x(16)
+       + 43.922173428509269 8D0*x(17) +
+      8.352989375082132 02D0*x(18) - 0.958821750950846804D0*x(19)
+       - 0.860246287496771303D0*x(20)
       y(15) = y(15) - 78.9585290740215004D0*x(21) +
+      43.922173428509269 8D0*x(22) + 8.352989375082132 02D0*x(23)
+      - 1954.936245050993 99D0*x(24) -
+      2.434167026240218 02D0*x(25) - 2597.51251185656298D0*x(26)
+      + 37578.76249270933 02D0*x(27) -
+      4.172084012088732 03D0*x(28) + 3.077920932009688 97D9*x(29)
+      - 41.06334439731767 99D0*x(35) +
+      3.573167209482252 97D-3*x(36)
       y(16) = -42.16668832387859 03D0*x(5) + 16.36587051303695D0*x(6)
+      - 453.129586109553202D0*x(7) + 351.460438213513001D0*x(8)
+      - 0.599754637061799697D0*x(9) -
+      4.632330831449649 02D-2*x(10) - 1.520116146415901D-2*x(11)
+      - 6.863707281844977 01D-2*x(12) +
+      0.483698505531560102D0*x(13) -
+      0.969166155996266099D0*x(14) +
+      0.519949869945893497D0*x(15) + 1.263028288468647 01D0*x(16)
+       - 1.740007095918280 01D0*x(17) -
+      1.189239579268164D-2*x(18) - 0.969166155996266099D0*x(19)
+      + 0.519949869945893497D0*x(20)
       y(16) = y(16) + 1.263028288468647 01D0*x(21) -
+      1.740007095918280 01D0*x(22) - 1.189239579268164D-2*x(23)
+      + 900.37846047579300 5D0*x(24) +
```

```
     +      0.643799490067852503D0*x(25) + 1238.144700088829O1D0*x(26)
     +       + 291.107672370571898D0*x(27) -
     +      9.359612011i78221493D-2*x(28) + 3.8160882301903249 7D7*x(29)
     +       - 0.652116307184986296D0*x(35) -
     +      2.3240999259453940 1D-5*x(36)
c------------------------------------------ l - u bounded limit
c  -- {nlarw2.evel limiter.4}
        y(17) = min( 0.017449999999999999D0, max(
     +            -0.017449999999999999D0, y(1) ) )
c------------------------------------------ state-space system
c  -- {nlarw2.controller.1}
c  -- ss  ab --
        xd(1) = -27.7971207546241899D0*x(1) - 25.4156325927014D0*x(2)
     +      + 0.96687429703138 1502D0*x(3) -
     +      8.71552238769890408D-2*x(4) - 9.51654087303377394D-3*y(4)
     +      - 1.68032371405914D-2*y(5)
        xd(2) = -6.876026223639343 02D0*x(1) -
     +      7.367110562937085 99D0*x(2) - 8.27135760534558596D-3*x(3)
     +      + 0.976139521572193303D0*x(4) -
     +      2.637988492486240 01D-3*y(4) + 1.501039866773363D-2*y(5)
        xd(3) = -52.380424882711 6199D0*x(1) -
     +      35.589941048672470 1D0*x(2) - 0.290086695207493903D0*x(3)
     +      - 0.218148213265887D0*x(4) - 7.090949693903226 04D-3*y(4)
     +      - 9.458798380241606 03D-3*y(5)
        xd(4) = -4067.666992244397 99D0*x(1) -
     +      5071.33417368828702D0*x(2) - 5.043376280250499D0*x(3) -
     +      17.02535819689388 01D0*x(4) - 1.157405145983781 01D-2*y(4)
     +      + 1.73620839739127D-2*y(5)
c------------------------------------------ state-space system
c  -- {nlarw2.arw2 moD.3}
c  -- ss  ab --
        xd(5) = x(10)
        xd(6) = x(11)
        xd(7) = x(12)
        xd(8) = x(13)
        xd(9) = -125.2459238677379 01D0*x(5) -
     +      45.033504131935390 4D0*x(6) - 117.872130364775799D0*x(7) +
     +      3906.630616470837 03D0*x(8) - 1.264140877762529 01D0*x(9) -
     +      0.127329745786802 6D0*x(10) - 0.200158294273304 4D0*x(11) +
     +      0.146365254176645 4D0*x(12) + 1.159698270631543 99D0*x(13)
     +      - 0.969152580431533295D0*x(14) +
     +      8.27755253907873397D-4*x(15) +
     +      6.043889655949902 02D-3*x(16) -
     +      6.856145165920952 01D-3*x(17) +
     +      5.018070223107642 95D-4*x(18) -
     +      0.969152580431533295D0*x(19) +
     +      8.27755253907873397D-4*x(20)
        xd(9) = xd(9) + 6.043889655949902 02D-3*x(21) -
     +      6.856145165920952 01D-3*x(22) +
     +      5.018070223107642 95D-4*x(23) + 1223.061625962262 01D0*x(24)
     +       + 0.899244109822731702D0*x(25) +
     +      1763.759691910593 01D0*x(26) + 698.1610555553561D0*x(27) -
     +      0.1264279353258893D0*x(28) + 6.716669502236605 06D7*x(29)
     +      - 1.867833656108772D0*x(35) + 7.655623225713645 98D-5*x(36)
```

```
+      - 4.84533115551499498D-4*u
   xd(10) = -31.7973764346982102D0*x(5) - 16.6021798016908D0*x(6)
+      - 135.2985761432064D0*x(7) + 2147.94963514691301D0*x(8) -
+      1.24869739759363099D0*x(9) - 0.2655459480335534D0*x(10) -
+      0.15325379235389659 9D0*x(11) - 1.050858874417987D-2*x(12)
+      + 1.278272593746799D0*x(13) - 1.27530495555126799D-3*x(14)
+      - 5.79125135412297698D0*x(15) +
+      2.918827546093392D-3*x(16) - 1.20106974790340501D-3*x(17)
+      + 3.667313377607812D-3*x(18) -
+      1.27530495555126799D-3*x(19) - 5.79125135412297698D0*x(20)
+      + 2.918827546093392D-3*x(21)
   xd(10) = xd(10) - 1.20106974790340501D-3*x(22) +
+      3.667313377607812D-3*x(23) + 4267.34662588543097D0*x(24)
+      + 3.5137358075365630 1D0*x(25) +
+      6539.00920066869003D0*x(26) + 779.8473335337912D0*x(27) -
+      0.1006533599710573D0*x(28) + 5.69431331591210403D7*x(29)
+      - 0.5308573137457699 01D0*x(35) +
+      6.995781207806434 95D-5*x(36) - 4.427709625193978D-4*u
   xd(11) = -4035.453860038237D0*x(5) - 5039.80498819288903D0*x(6)
+      - 10315.3494347062699D0*x(7) + 2.422344823719 04D5*x(8) -
+      27.902935086507 4499D0*x(9) - 5.004839619993 64602D0*x(10)
+      - 16.919934275288 0302D0*x(11) -
+      1.217457408188 24499D0*x(12) + 75.0144208438646292D0*x(13)
+      + 8.243002362753327 08D-3*x(14) +
+      1.939657213813227 99D-3*x(15) - 152.1134291940743D0*x(16)
+      - 5.803865414252263 03D-2*x(17) +
+      5.255237649770827 96D-2*x(18) +
+      8.243002362753327 08D-3*x(19) +
+      1.939657213813227 99D-3*x(20)
   xd(11) = xd(11) - 152.1134291940743D0*x(21) -
+      5.803865414252263 03D-2*x(22) +
+      5.255237649770827 96D-2*x(23) - 3324.495306375349D0*x(24)
+      - 3.661546561775992 03D0*x(25) -
+      3889.287973548708 01D0*x(26) + 70121.2766765817105D0*x(27)
+      - 16.230958826885 0602D0*x(28) +
+      8.243121322112151 98D9*x(29) - 58.8147948818334498D0*x(35)
+      + 6.037748827242839 02D-3*x(36) -
+      3.821360017242314 99D-2*u
   xd(12) = 3871.75335549381299D0*x(5) + 1535.837069757428D0*x(6)
+      - 34825.31679885881 02D0*x(7) - 1.034283535741036 99D5*x(8)
+      + 26.295468997189909 8D0*x(9) + 1.250989170291227D0*x(10)
+      + 2.630979503694448D0*x(11) - 18.3240096889994701D0*x(12)
+      + 9.726785776101678 05D0*x(13) -
+      4.201120788159818 04D-3*x(14) +
+      1.353056510651707 01D-3*x(15) +
+      4.063760124791731 99D-3*x(16) - 140.368755107959299D0*x(17)
+      + 0.141684134413155 902D0*x(18) -
+      4.201120788159818 04D-3*x(19) +
+      1.353056510651707 01D-3*x(20)
   xd(12) = xd(12) + 4.063760124791731 99D-3*x(21) -
+      140.368755107959299D0*x(22) + 0.141684134413155902D0*x(23)
+      + 2449.122483159866 99D0*x(24) +
+      2.205299818345310 03D0*x(25) + 1948.391105733950 01D0*x(26)
+      + 20503.23546981124 99D0*x(27) -
```

```
     +      8.71772134128349308D0*x(28) + 3.55976045603601098D9*x(29)
     +      + 56.3946108862526199D0*x(35) -
     +      3.6337807673383630 2D-3*x(36) + 2.29986124515086901D-2*u
        xd(13) = -463.806180072810101D0*x(5) -
     +      349.736890131978697D0*x(6) - 1195.977582526502D0*x(7) -
     +      40097.8732828805196D0*x(8) - 1.46494257127194799D0*x(9) +
     +      0.183876792527185799D0*x(10) -
     +      0.555117409039766599D0*x(11) +
     +      0.167101544533691599D0*x(12) - 29.8328022920818499D0*x(13)
     +       - 3.8129056653498640 2D-3*x(14) +
     +      5.78150706319158894D-4*x(15) - 0.1002548480818097D0*x(16)
     +      + 9.994206021040908D-2*x(17) - 6.49269676792184203D0*x(18)
     +       - 3.8129056653498640 2D-3*x(19) +
     +      5.78150706319158894D-4*x(20)
        xd(13) = xd(13) - 0.1002548480818097D0*x(21) +
     +      9.994206021040908D-2*x(22) - 6.49269676792184203D0*x(23)
     +      + 1019.796982139836D0*x(24) + 0.97690500705910919 6D0*x(25)
     +       + 2093.94171462465602D0*x(26) -
     +      5232.10908006390696D0*x(27) - 1.296059064581286D0*x(28) +
     +      1.285701909877968D8*x(29) - 7.11996318155942698D0*x(35) +
     +      6.16372950518372805D-4*x(36) - 3.90109462353402403D-3*u
        xd(14) = 0.1284819999999999 9D0*x(9) -
     +      40.173600000000080 4D0*x(10) + 4.33304999999998597D0*x(11)
     +      - 98.3560999999999694D0*x(12) + 1044.980000000003D0*x(13)
     +      - 84.25426553046417 94D0*x(14) -
     +      146.225000000000039 9D0*x(25) + 158.846999999999799D0*x(28)
     +      + 8.52704296673918397D-2*x(35) +
     +      4.06559376838870898D-2*x(36) - 0.2573160612904246D0*u
        xd(15) = -19.285300000000010 1D0*x(9) -
     +      5.569919999999999599D0*x(10) + 10.0029999999999899D0*x(11)
     +      - 29.131200000000040 2D0*x(12) -
     +      92.8560999999999694D0*x(13) - 84.25426553046417 94D0*x(15)
     +      - 111.353000000000009 9D0*x(25) +
     +      17.2043999999999602D0*x(28) + 4.65810662194168197D-2*x(35)
     +       + 2.22093043608741999D-2*x(36) -
     +      0.14056521747388740 1D0*u
        xd(16) = 6.897210000000000 95D0*x(9) -
     +      6.320390000000000 295D0*x(10) - 5.16849999999999499D0*x(11)
     +      + 2.076999999999999 801D0*x(12) +
     +      205.770999999999699D0*x(13) - 84.25426553046417 94D0*x(16)
     +      + 19.065199999999999 9D0*x(25) -
     +      59.310699999999999 8D0*x(28) + 3.427498257283779D-4*x(35)
     +      + 1.6341908455639 98D-4*x(36) - 1.03429800352151 9D-3*u
        xd(17) = -7.4448199999999993D0*x(9) +
     +      6.893470000000000 798D0*x(10) + 8.567650000000001 509D0*x(11)
     +      + 2.0852D0*x(12) - 145.588999999999899D0*x(13) -
     +      84.25426553046417 94D0*x(17) - 15.727300000000001D0*x(25) -
     +      32.450399999999950 4D0*x(28) + 3.19475760574534596D-2*x(35)
     +       + 1.523222841035782D-2*x(36) -
     +      9.64065089263157499D-2*u
        xd(18) = -13.32389999999999 801D0*x(9) -
     +      18.165200000000030 2D0*x(10) - 118.12700000000000 1D0*x(11)
     +      + 134.614999999999799D0*x(12) -
     +      949.754000000000801D0*x(13) - 84.25426553046417 94D0*x(18)
```

```
+      + 3.17905000000000398D0*x(25) -
+      397.355999999999803D0*x(28) + 0.2788932280042591D0*x(35)
+      + 0.13297301001563080 1D0*x(36) -
+      0.841601329212856797D0*u
      xd(19) = 82.220800000000000503D0*x(9) +
+      38.144600000000000804D0*x(10) - 9.01630999999997607D0*x(11)
+      + 31.33159999999999799D0*x(12) +
+      656.121999999999403D0*x(13) - 168.6879863655722D0*x(19) +
+      431.763000000000801D0*x(25) + 123.543000000000101D0*x(28)
+      + 0.78351950797469879 8D0*x(35) +
+      0.373572883525770998D0*x(36) - 2.364385338770716 99D0*u
      xd(20) = 2.0109900000000000699D0*x(9) +
+      29.276200000000000202D0*x(10) - 12.3716999999999799D0*x(11)
+      + 20.673900000000000002D0*x(12) +
+      247.386999999999698D0*x(13) - 168.6879863655722D0*x(20) +
+      264.549000000000902D0*x(25) + 26.5588999999999902D0*x(28)
+      + 1.0063890730269250 1D-2*x(35) +
+      4.798344701477913 04D-3*x(36) - 3.036927026251846D-2*u
      xd(21) = 22.594000000000005D0*x(9) - 5.377080000000000697D0*x(10)
+       + 8.38994999999999891D0*x(11) - 18.90300000000002D0*x(12)
+       + 142.49300000000004D0*x(13) - 168.6879863655722D0*x(21)
+      - 22.196900000000000301D0*x(25) +
+      168.935999999999702D0*x(28) + 0.223846017964695498D0*x(35)
+       + 0.106727147883002501D0*x(36) -
+      0.675488277740527096D0*u
      xd(22) = -23.138500000000000201D0*x(9) +
+      4.77663999999999 8601D0*x(10) - 12.28980000000001D0*x(11) +
+      13.89389999999999699D0*x(12) - 266.941999999999098D0*x(13)
+      - 168.6879863655722D0*x(22) + 19.15959999999995D0*x(25) -
+      43.065900000000000596D0*x(28) - 0.231364769970956501D0*x(35)
+       - 0.110312000383681901D0*x(36) +
+      0.6981772176182410 03D0*u
      xd(23) = 150.079999999999899D0*x(9) -
+      14.43409999999999999D0*x(10) + 184.738999999999599D0*x(11)
+      - 262.153000000000198D0*x(12) +
+      2774.570000000000698D0*x(13) - 168.6879863655722D0*x(23) +
+      28.312300000000000502D0*x(25) + 935.655999999999 8997D0*x(28)
+      + 0.302353929991241402D0*x(35) +
+      0.144158796714765301D0*x(36) - 0.912397447561808406D0*u
      xd(24) = x(25)
      xd(25) = -3.9478417999999997 02D5*x(24) -
+      1256.637000000000199D0*x(25) + 7.89568359999999404D6*x(26)
      xd(26) = -20.0D0*x(26) + y(17)
      xd(27) = x(28)
      xd(28) = -3.348D5*x(27) - 818.4000000000001498D0*x(28) +
+      7.201548D+11*x(29)
      xd(29) = x(30)
      xd(30) = -2.151D6*x(29) - 540.099999999998502D0*x(30) +
+      2.191D5*x(31)
      xd(31) = x(32)
      xd(32) = -2.191D5*x(31) - 185.3000000000002D0*x(32) +
+      3.742D6*x(33)
      xd(33) = x(34)
      xd(34) = -3.742D6*x(33) - 1446.5D0*x(34) + y(3)
```

```
      xd(35) = -0.3399999999999999D0*x(35) -
     +     0.16210799999999991D0*x(36) + 1.0260000000000299D0*u
      xd(36) = -0.35999999999999397D0*x(36) + u
c-----------------------------------------------------------------
c
      return
      end
```

Appendix D - MODEL.INP file for the ARW-2 model

This is an example of the input file, MODEL.INP, that must be created by the user. The odd numbered lines are dummy variables used to name the fields and the even numbered lines contain the actual data. This is the input file used to generate the results shown in figure 5.

```
model
arw2 flexible nonlinear model, mach .86, altitude 24k ft
case
maximize output #6
nstates  nout noutmx
36      17  6
tmaximp  deltat
10.0    0.005
sigma    akmin    akmax    nkvals
1530.0   10.0    15000.0   9
iouttype impres  iexcres  iwave
3         1         1        1
standard filename (iouttype=1)
arw1530.out
matrixx filename (iouttype=2)
arw1530.mat
```

Where these input quantities are defined:

model	A character variable describing the model used in the analysis
case	A character variable describing the case
nstates	Number of states in the model
nout	Number of output quantities used in the model
tmaximp	The length of the simulation used in the impulse responses
deltat	The time step used in the simulations
noutmx	The number of the output quantity to be maximized
sigma	The gust intensity used in the analysis, units of velocity
akmin	The minimum k value to be used in the analysis
akmax	The maximum k value to be used in the analysis
nkvals	The total number of k values to be used in the analysis 1, akmin is used 2, akmin and akmax are used ≥3, k values distributed logrithimically between akmin and akmax
iouttype	Specifies the format for the output 1, Normal ascii file 2, MATRIXχ readable ascii file 3, Both 1 and 2
impres	If 1, write the impulse responses to the output file(s)
iexcres	If 1, write the excitation waveform responses to the output file(s)
iwave	If 1, write the excitation waveform(s) to the output file(s)

Appendix E - A sample listing of the MFB1DS program output using the input file shown in apppendix D and the model listed in Appendix C

```
arw2 flexible nonlinear model, Mach .86, Altitude 24k ft
maximize output #6

 length of simulation for impulse responses =    10.00000
 number of time steps for impulse responses =   2001

 length of simulation for excitation responses =    20.0000
 number of time steps for excitation responses =   4001

 gust intensity =    1530.00

calculating impulse responses for each k value
calculating normalized excitation waveforms
   1  k =    10.00000  sqrt(energy) of output  6 =    568.177
   2  k =    24.9466   sqrt(energy) of output  6 =    1417.29
   3  k =    62.2333   sqrt(energy) of output  6 =    3536.37
   4  k =    155.251   sqrt(energy) of output  6 =    8820.35
   5  k =    387.298   sqrt(energy) of output  6 =    22003.6
   6  k =    966.177   sqrt(energy) of output  6 =    56134.6
   7  k =    2410.28   sqrt(energy) of output  6 =    162952.
   8  k =    6012.84   sqrt(energy) of output  6 =    509979.
   9  k =    15000.0   sqrt(energy) of output  6 =    1.49411E+06
calculating maximized responses
   1  k =    10.00000  maximum value of output  6 =    287000.
   2  k =    24.9466   maximum value of output  6 =    286965.
   3  k =    62.2333   maximum value of output  6 =    286988.
   4  k =    155.251   maximum value of output  6 =    286997.
   5  k =    387.298   maximum value of output  6 =    287025.
   6  k =    966.177   maximum value of output  6 =    289885.
   7  k =    2410.28   maximum value of output  6 =    296994.
   8  k =    6012.84   maximum value of output  6 =    279944.
   9  k =    15000.0   maximum value of output  6 =    249730.
saving data
```

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